compresses. At time $t = T/2$, the block reaches $x = -A$. Here the velocity and kinetic energy are equal to zero. The force on the block is $F = +kA$ and the potential energy stored in the spring is $U = \frac{1}{2}kA^2$. During the oscillations, the total energy is constant and equal to the sum of the potential energy and the kinetic energy of the system,

$$E_{\text{Total}} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2. \tag{15.12}$$

The equation for the energy associated with SHM can be solved to find the magnitude of the velocity at any position:

$$|v| = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}. \tag{15.13}$$

The energy in a simple harmonic oscillator is proportional to the square of the amplitude. When considering many forms of oscillations, you will find the energy proportional to the amplitude squared.

15.1 Check Your Understanding Why would it hurt more if you snapped your hand with a ruler than with a loose spring, even if the displacement of each system is equal?

15.2 Check Your Understanding Identify one way you could decrease the maximum velocity of a simple harmonic oscillator.

15.3 | Comparing Simple Harmonic Motion and Circular Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe how the sine and cosine functions relate to the concepts of circular motion
- Describe the connection between simple harmonic motion and circular motion

An easy way to model SHM is by considering uniform circular motion. **Figure 15.17** shows one way of using this method. A peg (a cylinder of wood) is attached to a vertical disk, rotating with a constant angular frequency. **Figure 15.18** shows a side view of the disk and peg. If a lamp is placed above the disk and peg, the peg produces a shadow. Let the disk have a radius of $r = A$ and define the position of the shadow that coincides with the center line of the disk to be $x = 0.00\,m$. As the disk rotates at a constant rate, the shadow oscillates between $x = +A$ and $x = -A$. Now imagine a block on a spring beneath the floor as shown in **Figure 15.18**.

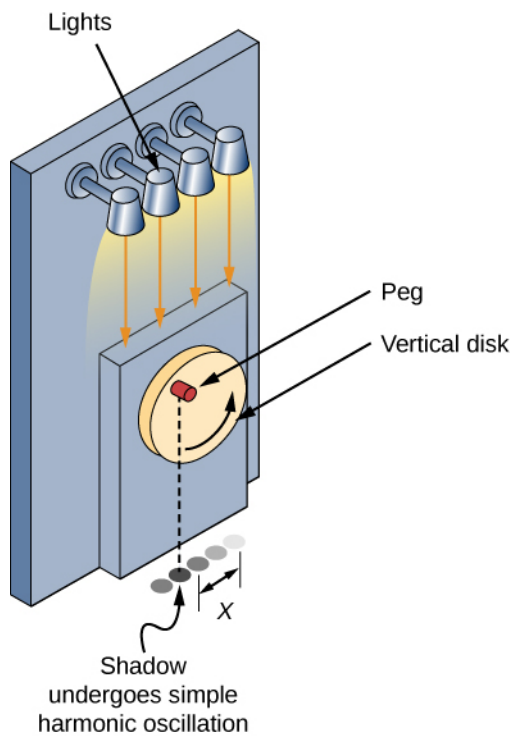

Figure 15.17 SHM can be modeled as rotational motion by looking at the shadow of a peg on a wheel rotating at a constant angular frequency.

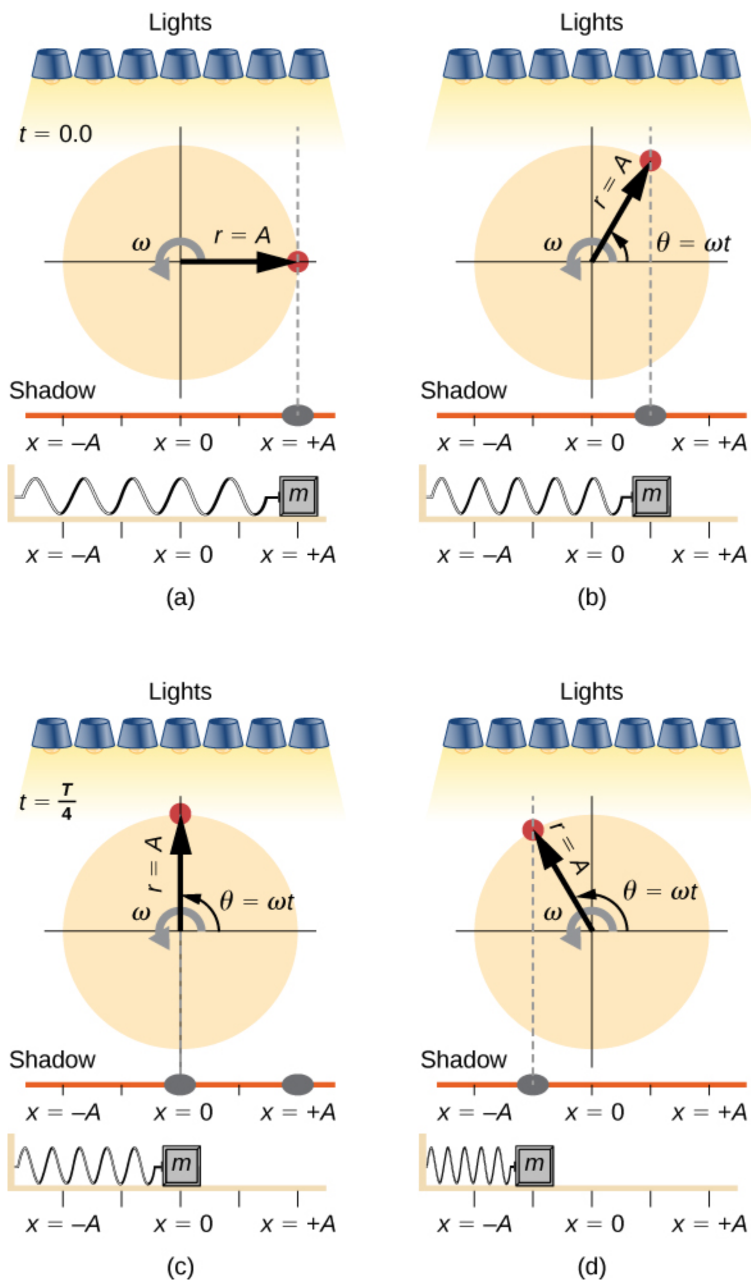

Figure 15.18 Light shines down on the disk so that the peg makes a shadow. If the disk rotates at just the right angular frequency, the shadow follows the motion of the block on a spring. If there is no energy dissipated due to nonconservative forces, the block and the shadow will oscillate back and forth in unison. In this figure, four snapshots are taken at four different times. (a) The wheel starts at $\theta = 0^o$ and the shadow of the peg is at $x = +A$, representing the mass at position $x = +A$. (b) As the disk rotates through an angle $\theta = \omega t$, the shadow of the peg is between $x = +A$ and $x = 0$. (c) The disk continues to rotate until $\theta = 90^0$, at which the shadow follows the mass to $x = 0$. (d) The disk continues to rotate, the shadow follows the position of the mass.

If the disk turns at the proper angular frequency, the shadow follows along with the block. The position of the shadow can be modeled with the equation

$$x(t) = A\cos(\omega t). \tag{15.14}$$

Recall that the block attached to the spring does not move at a constant velocity. How often does the wheel have to turn to have the peg's shadow always on the block? The disk must turn at a constant angular frequency equal to 2π times the frequency of oscillation $(\omega = 2\pi f)$.

Figure 15.19 shows the basic relationship between uniform circular motion and SHM. The peg lies at the tip of the radius, a distance A from the center of the disk. The x-axis is defined by a line drawn parallel to the ground, cutting the disk in half. The y-axis (not shown) is defined by a line perpendicular to the ground, cutting the disk into a left half and a right half. The center of the disk is the point $(x = 0,\ y = 0)$. The projection of the position of the peg onto the fixed x-axis gives the position of the shadow, which undergoes SHM analogous to the system of the block and spring. At the time shown in the figure, the projection has position x and moves to the left with velocity v. The tangential velocity of the peg around the circle equals $\bar{v}_{max}$ of the block on the spring. The x-component of the velocity is equal to the velocity of the block on the spring.

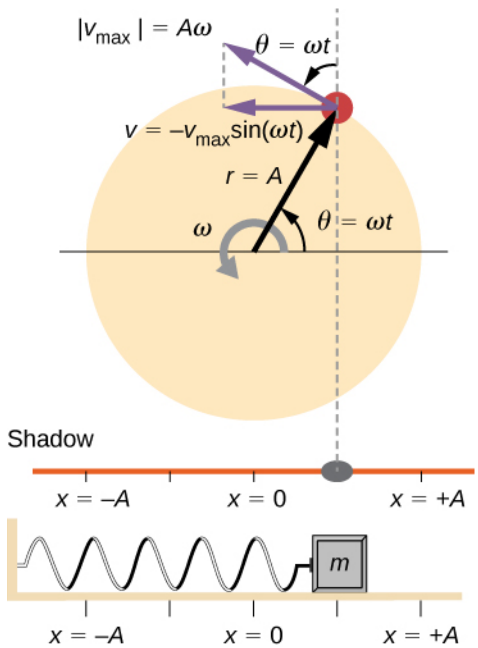

Figure 15.19 A peg moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes SHM. Also shown is the velocity of the peg around the circle, v_{max}, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle.

We can use Figure 15.19 to analyze the velocity of the shadow as the disk rotates. The peg moves in a circle with a speed of $v_{max} = A\omega$. The shadow moves with a velocity equal to the component of the peg's velocity that is parallel to the surface where the shadow is being produced:

$$v = -v_{max}\sin(\omega t). \tag{15.15}$$

It follows that the acceleration is

$$a = -a_{max}\cos(\omega t). \quad (15.16)$$

15.3 Check Your Understanding Identify an object that undergoes uniform circular motion. Describe how you could trace the SHM of this object.

15.4 | Pendulums

Learning Objectives

By the end of this section, you will be able to:

- State the forces that act on a simple pendulum
- Determine the angular frequency, frequency, and period of a simple pendulum in terms of the length of the pendulum and the acceleration due to gravity
- Define the period for a physical pendulum
- Define the period for a torsional pendulum

Pendulums are in common usage. Grandfather clocks use a pendulum to keep time and a pendulum can be used to measure the acceleration due to gravity. For small displacements, a pendulum is a simple harmonic oscillator.

The Simple Pendulum

A **simple pendulum** is defined to have a point mass, also known as the pendulum bob, which is suspended from a string of length L with negligible mass (**Figure 15.20**). Here, the only forces acting on the bob are the force of gravity (i.e., the weight of the bob) and tension from the string. The mass of the string is assumed to be negligible as compared to the mass of the bob.

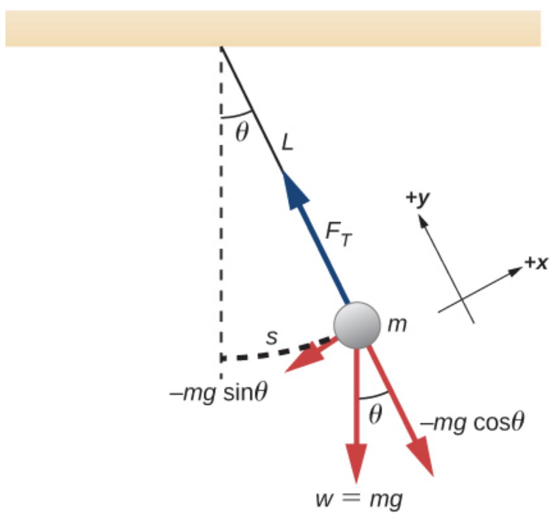

Figure 15.20 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg\sin\theta$ toward the equilibrium position—that is, a restoring force.

Consider the torque on the pendulum. The force providing the restoring torque is the component of the weight of the pendulum bob that acts along the arc length. The torque is the length of the string L times the component of the net force